Chapter 3

Methodology

3.1 Model

The model in this paper follows almost the same structure as Iacoviello and Neri (2008). In their paper, the economy is populated by patient households (lenders), impatient households (borrowers), housing sector, consumption goods sector and government. Households are heterogeneous that patient households receive utility from consumption and accumulating real estate, and provide loans to impatient households and capitals to firms. Impatient households receive loans from patient households and affected by house collateral. Firms contain two sectors that consumption goods sector produces final goods for households with monopolistic power and intermediate inputs for the housing sector. Housing sector produces new houses using labor, land, housing capital and intermediate inputs.

However, this model is different from Iacoviello (2008) model in three crucial ways. First, the government sector is introduced into the model, because this paper focus on the government policy analysis. Besides, the government expenditure is separated into consumption expenditure and investment expenditure for the real estate market, so we can see the impact of affordable housing policy on the real estate market. Second, the land policy is designed to simulate the China's real situation that land price is determined by land auction. Last, housing tax is introduced into the model to measure the effect of property tax policy. The model covers some Chinese features, so it has a better ability to match the Chinese situation, and thus stimulate the Chinese economy.

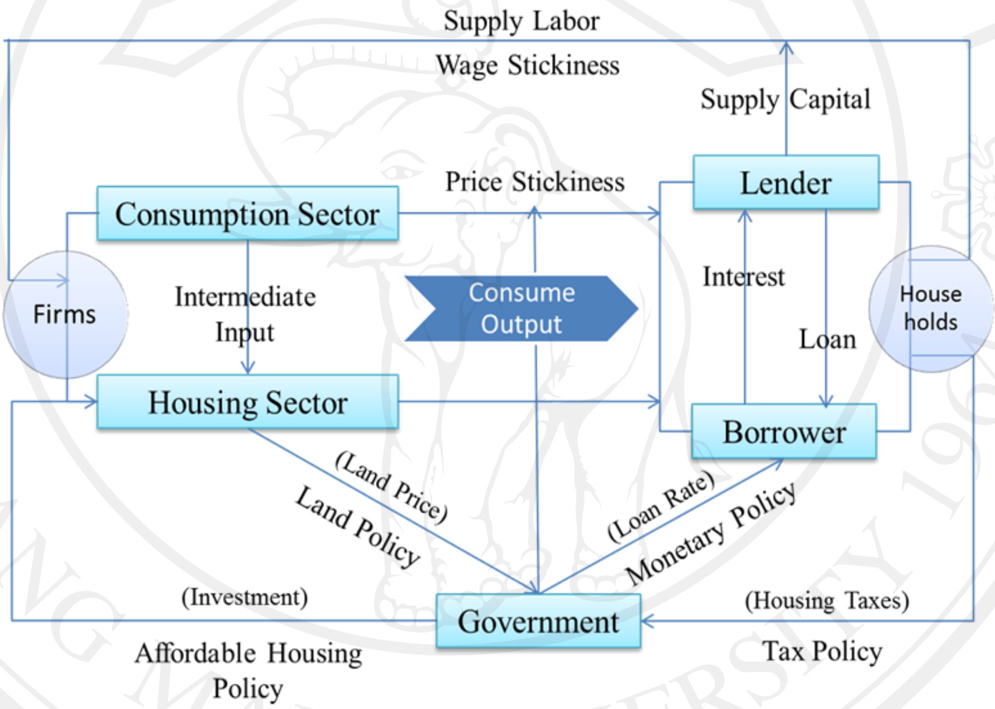

Figure 3.1 Structure of Model

3.1.1 Households

(1) Patient Households

The representative patient households (Lenders) are infinitely-lived and seek to maximize

$$E_0 \sum_{t=0}^{\infty} \left(\beta G_C\right)^t z_t \left(\Gamma_c \log\left(c_t - \varepsilon c_{t-1}\right) + j_t \log h_t - \frac{\tau_t}{1+\eta}\left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi}\right)^{\frac{1+\eta}{1+\xi}} \right) \tag{3.1}$$

where c, h, n_c, n_h represent consumption, housing, hours in the consumption sector and hours in the housing sector. β denotes the discount factor. E is the expectation operator and G_C is the gross growth rate of consumption along the balanced growth path. The scaling factors $\Gamma_c = (G_C - \varepsilon)/(G_C - \beta\varepsilon G_C)$ ensure that the marginal utilities of consumption are $1/c$ in the steady state. η measures the inverse Frisch elasticity of labor supply and ε is the degree of habit formation in consumption. The parameter ξ describes the inverse elasticity of substitution across hours. Random variations in z_t, j_t and τ_t capture shocks to intertemporal preferences, to the demand for housing and to the supply of labor respectively. These shocks follow the stochastic processes:

$$\ln z_t = \rho_z \ln z_{t-1} + \mu_{z,t}, \qquad \mu_{z,t} \sim N\left(0, \sigma_z\right) \tag{3.2}$$

$$\ln j_t = \left(1 - \rho_j\right) \ln \overline{j} + \rho_j \ln j_{t-1} + \mu_{j,t}, \qquad \mu_{j,t} \sim N\left(0, \sigma_j\right) \tag{3.3}$$

$$\ln \tau_t = \rho_\tau \ln \tau_{t-1} + \mu_{\tau,t}, \qquad \mu_{\tau,t} \sim N\left(0, \sigma_\tau\right) \tag{3.4}$$

where ρ_z, ρ_j, ρ_τ measure the persistence of those shocks, and $\mu_{z,t}$, $\mu_{j,t}$, $\mu_{\tau,t}$ are independent and identically distributed $(i.i.d)$.

Each period, patient households maximize their lifetime utility subject to the budget constraint:

$$\begin{aligned} & c_t + \frac{k_{c,t}}{A_{k,t}} - \left(\frac{1-\delta_{kc}}{A_{k,t}}\right) k_{c,t-1} + k_{h,t} - (1-\delta_{kh}) k_{h,t-1} + q_t h_t - (1-\delta_h) q_t h_{t-1} \\ & + b_t + k_{b,t} = \frac{w_{c,t}}{X_{wc,t}} n_{c,t} + \frac{w_{h,t}}{X_{wh,t}} n_{h,t} + R_{c,t} k_{c,t-1} + R_{h,t} k_{h,t-1} + p_{b,t} k_{b,t} \\ & + \frac{R_{t-1} b_{t-1}}{\pi_t} + f_t - \psi_{h,t} q_t h_t - \phi_t \end{aligned} \tag{3.5}$$

Patient households obtain wages from working in two sectors (working hours $n_{c,t}$ and $n_{h,t}$), lump-sum profits f_t from final good firms and labor unions, rental income from holding consumption capital $k_{c,t}$ and housing capital $k_{h,t}$, and interest from offering one-period loans b_{t-1} to impatient households. And then subject to this resource constraint, they choose plans for consumption c_t, intermediate goods $k_{b,t}$, one-period loans b_t and investment in consumption capital, housing capital and housing h_t (priced at q_t) to maximize their utility. The term $w_{h,t}$, $w_{c,t}$, $R_{c,t}$, $R_{h,t}$, δ_{kc}, δ_{kh} denote real wage, capital rental rates and depreciation rates in housing and consumption sector. R_t is riskless nominal return rate of loan. $A_{k,t}$ captures investment-specific technological shocks. $X_{wc,t}$ and $X_{wh,t}$ denote the makeup between the wage paid by the wholesale firm and the wage paid to the household, which accrues to the labor unions. $\pi_t = P_t/P_{t-1}$ is the gross money inflation rate in the consumption sector. $\emptyset_t$ denotes convex adjustment costs for capital and ψ_h is the housing tax rate.

(2) Impatient Household

The representative impatient households (Borrowers) are expressed by the variables with a prime and their optimization problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \left(\beta' G_C\right)^t z_t \left(\Gamma'_c \log\left(c'_t - \varepsilon' c'_{t-1}\right) + j_t \log h'_t - \frac{\tau_t}{1+\eta'} \left(\left(n'_{c,t}\right)^{1+\xi'} + \left(n'_{h,t}\right)^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right) \quad (3.6)$$

subject to

$$c'_t + q_t h'_t - \left(1-\delta_h\right) q_t h'_{t-1} + \frac{R_{t-1}}{\pi_t} b'_{t-1} = \frac{w'_{c,t}}{X'_{wc,t}} n'_{c,t} + \frac{w'_{h,t}}{X'_{wh,t}} n'_{h,t} + b'_t + f'_t - \psi_{h,t} q_t h'_t \quad (3.7)$$

where

$$b'_t \leq m E_t \left(q_{t+1} h'_t \pi_{t+1} / R_t\right) \quad (3.8)$$

Impatient households do not accumulate capital and do not own final good firms (their lump-sum profits come only from labor unions) and their only form of wealth will be their houses. So they use wages from work ($\acute{w}_{c,t}$ and $\acute{w}_{h,t}$), labor unions' lump-sum profits $\acute{f}_t$ and loans $\acute{b}_t$ from patient households to make the decision of consumption, purchasing housing and paying for interest and property tax. In addition, the maximum amount they can borrow is affected by house collateral and the loan-to-value ratio is denoted as m.

(3) Wage Stickiness

Sticky wages are introduced into the labor market by assuming wage rigidity at the union level and the implicit costs of adjusting nominal wages following Calvo-style contracts that a fraction $1-\theta_{wi}$ of unions in each sector set wages optimally while others cannot do so in each period. The unions received labor services from patient and impatient households homogeneously at the wage w_t^w, differentiated labor services as in Smets and Wouters (2007), reassembled these services into the homogeneous labor composites n_c, n_h, $\acute{n}_c$, $\acute{n}_h$ and supplied them at a markup over the marginal cost to the firms. These assumptions deliver the following wage Phillips curves:

$$\omega_{c,t}-\iota_{wc}\log\pi_{t-1}=\beta G_C\left(E_t\omega_{c,t+1}-\iota_{wc}\log\pi_t\right)-\varepsilon_{wc}\log\left(X_{wc,t}/X_{wc}\right) \tag{3.9}$$

$$\omega'_{c,t}-\iota_{wc}\log\pi_{t-1}=\beta' G_C\left(E_t\omega'_{c,t+1}-\iota_{wc}\log\pi_t\right)-\varepsilon'_{wc}\log\left(X_{wc,t}/X_{wc}\right) \tag{3.10}$$

$$\omega_{h,t} - \iota_{wh} \log \pi_{t-1} = \beta G_C \left(E_t \omega_{h,t+1} - \iota_{wh} \log \pi_t \right) - \varepsilon_{wh} \log \left(X_{wh,t} / X_{wh} \right) \tag{3.11}$$

$$\omega'_{h,t} - \iota_{wh} \log \pi_{t-1} = \beta' G_C \left(E_t \omega'_{h,t+1} - \iota_{wh} \log \pi_t \right) - \varepsilon'_{wh} \log \left(X_{wh,t} / X_{wh} \right) \tag{3.12}$$

where $\omega_{i,t} = w_{i,t} - w_{i,t-1} + \pi_t$ denotes nominal wage inflation for each sector-household pair. ι_{wi} represents the elasticity between index wage and the previous period inflation rage in each sector. $\varepsilon_{wc} = (1-\theta_{wc})(1-\beta G_C \theta_{wc})/\theta_{wc}$, $\acute{\varepsilon}_{wc} = (1-\theta_{wc})\left(1-\acute{\beta} G_C \theta_{wc}\right)/\theta_{wc}$, $\varepsilon_{wh} = (1-\theta_{wh})(1-\beta G_C \theta_{wh})/\theta_{wh}$ and $\acute{\varepsilon}_{wh} = (1-\theta_{wh})\left(1-\acute{\beta} G_C \theta_{wh}\right)/\theta_{wh}$

3.1.2 Firms

(1) Consumption Goods Sector

The wholesale firms hire labor and rent capital to produce intermediate goods Y_t in order to maximize profits:

$$\max \frac{Y_t}{X_t} - \left(w_{c,t} n_{c,t} + w'_{c,t} n'_{c,t} + R_{c,t} k_{c,t-1} \right) \tag{3.13}$$

Intermediate goods (nominal price P_t^w) are then transformed into final goods (priced at P_t) from final goods firms. So we define $X_t = P_t / P_t^w$ as the markup of final over intermediate goods. The production technology is:

$$Y_t = \left(A_{c,t} \left(n_{c,t}^{\alpha} n_{c,t}'^{1-\alpha} \right) \right)^{1-\nu_c} k_{c,t-1}^{\nu_c} \tag{3.14}$$

where $A_{c,t}$ is a measure of productivity in the consumption goods sector. ν_c denotes outcome share of capital in consumption sector

We assume monopolistic competition in the consumption goods sector. The final goods firms buy wholesale goods Y_t from wholesale firms at the price P_t^w in a competitive market, differentiate the goods at no cost, set prices subject to a Calvo price and sell them to the two kinds of households. Under Calvo pricing with partial indexation to past inflation, the pricing rules set by the final goods firms imply the price Phillips curve that is isomorphic to the wage Phillips curve.

$$\log \pi_t - \iota_\pi \log \pi_{t-1} = \beta G_C \left(E_t \log \pi_{t+1} - \iota_\pi \log \pi_t \right) - \varepsilon_\pi \log \left(X_t / X \right) + \log \mu_{p,t} \tag{3.15}$$

where $\varepsilon_\pi = (1-\theta_\pi)(1-\beta G_C \theta_\pi)/\theta_\pi$. As in Smets and Wouters (2007), we allow for cost-push shocks that affect inflation independently from fluctuations in the real marginal cost. These shocks are assumed to be i.i.d. with zero mean and variance equal to σ_p^2.

(2) Housing Sector

In housing sector, firms solve the following problem:

$$\max q_t IH_t - \left(w_{h,t} n_{h,t} + w'_{h,t} n'_{h,t} + R_{h,t} k_{h,t-1} + p_{l,t-1} l_{t-1} + p_{b,t} k_{b,t} \right) \tag{3.16}$$

where

$$IH_t = \left(A_{h,t} \left(n_{h,t}^{\alpha} n_{h,t}'^{1-\alpha} \right) \right)^{1-\nu_h-\nu_b-\nu_l} \left(k_{h,t-1} + G_{i,t-1} \right)^{\nu_h} k_{b,t}^{\nu_b} l_{t-1}^{\nu_l} \tag{3.17}$$

The representative firm use labor (n_h and n'_h), capital in housing sector, k_h, government investment, $G_{i,t}$, land, l_t and the intermediate input, $k_{b,t}$ produced in consumption goods sector to produce new houses IH_t. The term $A_{h,t}$ is a measure of productivity in the housing sector. α denotes the labor income share of unconstrained households. ν_h, ν_b and ν_l represent the outcome share of capital, intermediate input and land.

Different from consumption goods sector, we rule out price rigidities in housing sector. The reason is that: First, most new houses are priced for the first time when they are sold. Second, housing firms cannot increase the housing price after they priced according to China's new real estate policy.

3.1.3 Government

(1) Affordable Housing Policy and Tax Policy

To implement an affordable housing policy and tax policy, every period, the government collects housing taxes $\psi_{h,t} q_t (h_t + h_t')$ from the two kinds of households, earns land income $p_{l,t-1} l_{t-1}$ by providing land to housing firms in last period, spends $G_{e,t}$ on purchasing goods in the consumption sector, and invests $G_{i,t}$ in the housing sector as capital input. The government budget constraint is:

$$G_t = \psi_{h,t} q_t \left(h_t + h_t'\right) + p_{l,t-1} l_{t-1} \tag{3.18}$$

$$G_{i,t} = \left(1 - \theta_g\right) G_t + g_t \tag{3.19}$$

$$G_{e,t} = \theta_g G_t \tag{3.20}$$

$$\ln g_t = \left(1 - \rho_g\right) \ln \bar{g} + \rho_g \ln g_{t-1} + \mu_{g,t} \tag{3.21}$$

where G_t is the total government expenditure, $G_{i,t}$ and $G_{e,t}$ represent the government expenditure in housing sector and consumption goods sector. $\psi_{h,t}$ is property tax rate. g_t captures an affordable housing expenditure. The term $\mu_{g,t}$ captures a zero-mean, i.i.d. affordable housing policy shock with variance σ_g^2.

For tax policy, when the government increases the property tax rate, the households need to spend more money on holding houses, which will decrease their preference of purchasing houses, thus curb speculation. The function of housing tax shock is shown as follows:

$$\ln\psi_{h,t} = (1-\rho_h)\ln\bar{\psi} + \rho_h \ln\psi_{h,t-1} + \mu_{h,t} \tag{3.22}$$

(2) Land Policy

The land policy in China is different from other countries. First, the land is publicly owned, not private, and the land income is an important source of government revenues. Second, the land price is determined by auction. So we assume land price is stochastic to capture the feature of land policy.

$$p_{l,t} = p_{l,t-1}^{\rho_{pl}} \frac{\mu_{pl,t}}{e_t} \tag{3.23}$$

where $p_{l,t}$ is price of land for housing sector, ρ_{pl} is the smoothing parameter, and the term $\mu_{pl,t}$ captures independent and identically distributed postitive land cost shock with mean zero and variance σ_{pl}^2, while e_t is a stochastic process in order to model the new land policy.

$$\ln e_t = \rho_e \ln e_{t-1} + \mu_{e,t} \tag{3.24}$$

The land policy is aimed to decline land cost by increasing security money of land auction, curbing land hoarding and testing the utilization ratio of land, thus the land cost will stay in a normal level. With lower land cost, the housing firms can produce more houses and make housing prices down.

(3) Monetary Policy

About the monetary policy, the People Bank of China has never published its monetary policy model, but some researchers have suggested that China's monetary policy rule is close to a Taylor's rule (Yuan, 2008; Zhang, 2009). So in this study, we assume that the government follows a Taylor rule which responds to inflation and GDP growth to adjust the interest rate R_t:

$$R_t = R_{t-1}^{r_R} \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{G_C GDP_{t-1}} \right)^{(1-r_R)r_Y} \overline{rr}^{1-r_R} \mu_{R,t} \tag{3.25}$$

where $\overline{rr}$ is the steady-state real interest rate (which we assume to be equal to $1/\beta$, the patient households discount rate), The term $\mu_{R,t}$ which captures a monetary policy shock follows a normal distribution with zero mean and finite standard deviation σ_R.

In monetary policy, impatient households' amount of loan is affected by interest rate. Once the interest rate increase, the loan amount of borrowers will decrease, which can limit purchase of housing, and then cool the real estate market.

3.1.4 Market Equilibrium

There are three markets in the model. The goods market produces consumption (households and government), investment in two sectors and intermediate inputs for the housing market. The housing market produces new homes for the two kinds of households. In the loan market, impatient agents rent one-period collateralized nominal loans from patient households. The three market clearing conditions are:

$$C_t + IK_{c,t} / A_{k,t} + IK_{h,t} + G_{e,t} + k_{b,t} = Y_t - \phi_t \tag{3.26}$$

$$h_t + h'_t - (1-\delta_h)(h_{t-1} + h'_{t-1}) = IH_t \tag{3.27}$$

$$b_t = b'_t \tag{3.28}$$

where $C_t = c_t + \acute{c}_t$ is aggregate consumption, $IK_{c,t} = k_{c,t} - (1 - \delta_{kc})k_{c,t-1}$ and $IK_{h,t} = k_{h,t} - (1 - \delta_{kh})k_{h,t-1}$ are the two components of business investment, expressed in real units. $G_{e,t}$ is government expenditure.

3.2 Research Methods

The process starts from model construction to obtain a set of first order condition by solving the problems of the optimizing agent with the Lagrange multipliers method. After that, the perturbation method is applied to solve the model and find out the impulse response function.

The parameters of the model are estimated by calibration and Bayesian method which obtains the posterior distribution of the parameters on the basis of likelihood function and prior distribution.

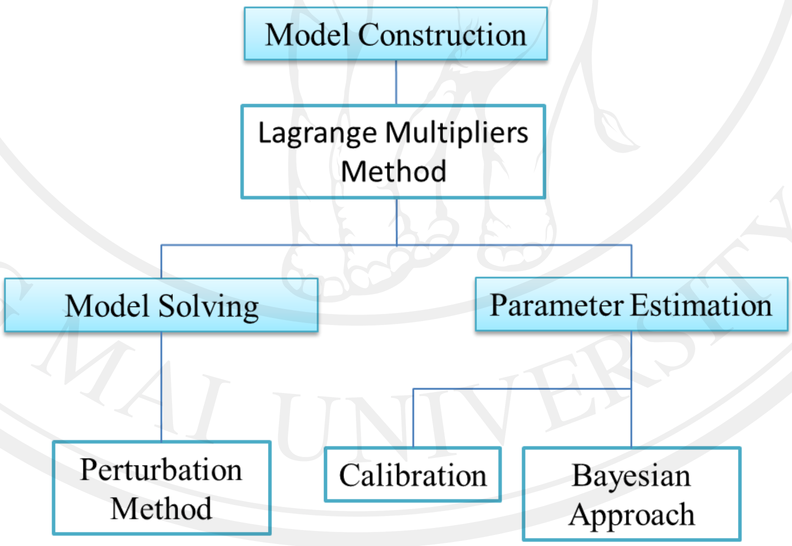

Figure 3.2 Structure of Research Method

3.2.1 Perturbation Method

(1) Writing the model in a general form

First, the first order and equilibrium conditions of DSGE model will be written into the following general form:

$$E_t\left[f\left(y_{t+1}, y_t, y_{t-1}, u_t\right)\right] = 0 \tag{3.29}$$

$$E\left(u_t\right) = 0 \tag{3.30}$$

$$E\left(u_t u_t'\right) = \Sigma_u \tag{3.31}$$

where y_t is a vector of endogenous variables of any dimension, and u_t denotes a vector of exogenous stochastic shocks of any dimension.

The solution to this system is what we call the policy function. It is a set of equations relating variables in the current period to the past state of the system and shocks. The policy function can be shown as:

$$y_t = g\left(y_{t-1}, u_t\right) \tag{3.32}$$

Using this function to rewrite the system in terms of past variables, and current and future shocks:

$$F\left(y_{t-1}, u_t, u_{t+1}\right) = f\left(g\left(g\left(y_{t-1}, u_t\right), u_{t+1}\right), g\left(y_{t-1}, u_t\right), y_{t-1}, u_t\right) \tag{3.33}$$

$$E_t\left[F\left(y_{t-1},u_t,u_{t+1}\right)\right]=0 \tag{3.34}$$

Ideally, the model solution would be obtained directly from analytical manipulation of those equations. However, these are highly non-linear, which makes it difficult for us to find the exact expression for $g(.)$. Thus, we need to perform a tractable linear approximation next.

(2) Computing the steady-state of the model

To perform the approximation, it is necessary to compute the steady-state of the model, which represents the situation where there are no innovations and variables are assumed to be a constant value in every period. Using $\bar{y}$ to denote the steady-state value of y_t, and the steady-state of the model is then given by:

$$f\left(\bar{y},\bar{y},\bar{y},0\right)=0 \tag{3.35}$$

$$\bar{y}=g\left(\bar{y},0\right) \tag{3.36}$$

(3) Solving the approximate linear model

The first order Taylor expansion around $\bar{y}$ is shown as:

$$
\begin{aligned}
&E_t\left[F^{(1)}\left(y_{t-1},u_t,u_{t+1}\right)\right]= \\
&E_t\left[f\left(\bar{y},\bar{y},\bar{y}\right)+f_{y+}\left(g_y\left(g_y\hat{y}+g_u u\right)+g_u u'\right)+f_{y0}\left(g_y\hat{y}+g_u u\right)+f_{y-}\hat{y}+f_u u\right] \\
&=0
\end{aligned} \tag{3.37}
$$

where $\hat{y} = y_{t-1} - \bar{y},\ u = u_t,\ \acute{u} = u_{t+1},\ f_{y+} = \frac{\partial f}{\partial y_{t+1}},\ f_{y0} = \frac{\partial f}{\partial y_t},\ f_{y-} = \frac{\partial f}{\partial y_{t-1}},$ $f_u = \frac{\partial f}{\partial u_t},\ g_y = \frac{\partial g}{\partial y_{t-1}},\ g_u = \frac{\partial g}{\partial u_t}$

The next step is to take expectations of the expansion, and obtain that:

$$
\begin{aligned}
&E_t\left[F^{(1)}\left(y_{t-1},u_t,u_{t+1}\right)\right]= \\
&\left[f\left(\bar{y},\bar{y},\bar{y}\right)+f_{y+}\left(g_y\left(g_y\hat{y}+g_u u\right)\right)+f_{y0}\left(g_y\hat{y}+g_u u\right)+f_{y-}\hat{y}+f_u u\right] \\
&=\left(f_{y+}g_y g_y + f_{y0}g_y + f_{y-}\right)\hat{y}+\left(f_{y+}g_y g_u + f_{y0}g_u + f_u\right)u \\
&=0
\end{aligned} \tag{3.38}
$$

The above equation shows that future shocks drop out when taking expectations of the linearized equations, because they only enter with their first moments (which are zero in expectations), This is the reason why certainty equivalence holds in a linearized system and its first order Taylor expansion. The second thing to note is that the two unknown variables g_y and g_u in the above equation will help us recover the policy function g.

Since the above equation holds for any $\hat{y}$ and u, each parenthesis must be null and we can solve each at a time. The first parenthesis generates a quadratic equation in g_y, which we can solve it by using a series of algebraic trics. Incidentally, one of the conditions that come out of the solution of this equation is the Blanchard-Kahn condition: there must be as many roots larger than one in modulus as there are forward-looking variables in the model. After obtaining g_y, we can get g_u straightforward from the second parenthesis.

Finally, notice that a first order linearization of the function g yields:

$$y_t = \bar{y} + g_y \hat{y} + g_u u \tag{3.39}$$

Since g_y and g_u have been recovered, we have solved for the (approximate) policy (or decision) function and have succeeded in solving DSGE model.

3.2.2 Bayesian Estimation

Bayesian estimation entails obtaining the posterior distribution of the model's parameters (θ), conditional on the data. According to Bayes theorem, the posterior distribution of the parameters is obtained by combining the likelihood function and the prior distribution as follows:

$$p(\theta|Y) \propto L(\theta|Y)\, p(\theta) \tag{3.40}$$

where $p(\theta|Y)$ and $p(\theta)$ denote the posterior and prior distribution of the parameters vector θ, and $L(\theta|Y)$ is the likelihood function for the observed data Y. The term $\propto$ represents proportionality.

Likelihood Function

To perform the estimation of the remaining parameters, using Bayesian approach, the first step is obtaining the likelihood function, which corresponds to the joint density of all variables in the data sample, conditional on the structure and parameters of our model. For this, the first thing is to rewrite the solution of a DSGE model as a system in the following manner:

$$y_t^* = M\bar{y}(\theta) + M\hat{y}_t + N(\theta)x_t + \eta_t \tag{3.41}$$

$$\hat{y}_t = g_y(\theta)\hat{y}_{t-1} + g_u(\theta)u_t \tag{3.42}$$

$$E(\eta_t\eta_t') = V(\theta) \tag{3.43}$$

$$E(u_t u_t') = Q(\theta) \tag{3.44}$$

where $\hat{y}_t$ are variables in deviations from steady state, $\bar{y}$ is the vector of steady state values and θ the vector of deep (or structural) parameters to be estimated.

In first equation, only y_t^* is observable, and it is related to the true variables with an error η_t. The term $N(\theta)x_t$ capture the trend depends on the deep parameters. The second equation expresses a relationship among true endogenous variables that are not directly observed.

The next step is to estimate the likelihood of the DSGE solution system. The equations are linear in the endogenous and exogenous variables, so the likelihood can be evaluated with the Kalman filter.

For $t = 1,\ldots,T$ and with initial values y_1 and P_1 given, the recursion follows

$$v_t = y_t^* - \bar{y}^* - M\hat{y}_t - Nx_t \tag{3.45}$$

$$F_t = MP_tM' + V \tag{3.46}$$

$$K_t = g_y P_t g_y' F_t^{-1} \tag{3.47}$$

$$\hat{y}_{t+1} = g_y \hat{y}_t + K_t v_t \tag{3.48}$$

$$P_{t+1} = g_y P_t \left(g_y - K_t M\right)' + g_u Q g_u' \tag{3.49}$$

From the Kalman filter recursion, the log-likelihood we got is given by

$$\ln L\left(\theta^{*}\middle|Y_{T}^{*}\right)=-\frac{Tk}{2}\ln\left(2\pi\right)-\frac{1}{2}\sum_{t=1}^{T}\left|F_{t}\right|-\frac{1}{2}v_{t}'F_{t}^{-1}v_{t} \tag{3.50}$$

where the vector θ^* contains the parameters we have to estimate: $\theta,\ V(\theta)$ and $Q(\theta)$ and Y_T^* represents the set of observable endogenous variables y_t^* found in the measurement equation.

3.3 Description of Data

Ten macroeconomic quarterly series over the first quarter of 1999 to the last quarter of 2011: consumption, real estate investment, capital investment, labor in consumption goods sector, labor in housing sector, house price, interest rate, inflation, wage in consumption goods sector and wage in housing sector are used in the study. Inflation is measured by using the quality-adjusted CPI index. The capital investment is represented by fixed-asset investment. The National Housing Sensitive index is used as the measure of the real house price and the 1-year loan interest rate is applied to capture the loan interest rate. All variables are from the Statistics Bureau of China and already being tested to be stationary by using log difference and HP filter following standard practice.

3.4 Calibration

The basic idea of calibration is to select parameter values according to microeconomic evidence and then compare the model's predictions about the variance and covariance of various series with those in the data.

A number of parameters in this study are excluded from the estimation and need to be calibrated. Because they are either notoriously difficult to estimate or better be identified by using other information suggested by Iacoviello and Neri (2008). The discount factor for patient households is fixed at 0.984, as in Li et al. (2011), implying a steady state real interest rate of around 6 percent on an annual basis and we choose a value 0.97 for impatient households to guarantee a large enough impatience motive. The parameter of housing preference is set to 0.16 in order to match the real estate share of GDP in China. Following Wang and Zhu (2009), the capital share in consumption sector is fixed at 0.436. In housing sector, we set the share of housing capital, land and intermediate goods are all equal to 0.2. Markups are set to 1.15 in both goods markets and labor market (in each sector). The depreciation rate for housing is equal to 0.01, corresponding to an annual discount rate of 4%, whereas the depreciation rate of consumption capital and housing capital are set to 0.025 and 0.03. The loan-to-value ratio is taken from Xiao and Peng (2011), and the share of consumption expenditure in government revenues is fixed at 0.95 according to the actual time series data of China's economy.

Table 3.1 Calibration

Parameter	Value	Description
$\beta,\ \beta'$	0.984, 0.97	Subjective discount factor
$X,\ X_{wc,t},\ X_{wh,t}$	1.15	Markup of gross price and wage
j	0.16	Housing preference
$\nu_c,\ \nu_h$	0.436, 0.2	Capital share in both sectors
$\nu_b,\ \nu_l$	0.2	Share of land and intermediate goods
$\delta_h,\ \delta_{kc},\ \delta_{kh}$	0.01, 0.025, 0.03	Depreciation of housing and capital
m	0.7	Loan-to-value ratio
θ_g	0.95	Share of consumption expenditure

3.5 Prior Distributions

The next step is the specification of the prior distributions, $p(\theta)$. Each prior is a probability density function of a parameter, constituting a formal way of specifying probabilities to the values that parameters can assume, based on past studies or/and occurrences or simply reflecting subjective views of the researcher. It is a representation of belief in the context of the model, set without any reference to the data, constituting an additional, independent, source of information.

The prior's functional form is specified according to each parameter's characteristics, which means that: inverse gamma distribution for parameters bounded to be positive; beta distribution for parameters bounded between zero and one and normal distribution for non-bounded parameters.

Normally, there are two strategies for defining prior distribution: the first one is to determine the priors based on the existing empirical evidence and their implications for macroeconomic dynamics; the second one is to set priors with reasonable means and a large support so that the distribution can cover a considerable range of parameter values.

The standard errors of the shocks are assumed to follow a uniform distribution with a mean of 3 and 1.7 degrees of freedom, while the persistence parameters are beta distributed with mean 0.8 and standard deviation 0.1. For the parameters of monetary policy which is based on a standard Taylor rule, we follow Sun and Sen (2010), the interest rate smoothing parameter is set as a beta distribution with mean 0.75 and standard errors 0.1, and the parameters capturing the response to changes in inflation and output gap are described by a Normal distribution around a mean of 1.63 and 0.01 with a standard error of 0.05.

The parameters in utility functions are assumed to be distributed as follows. The habit parameters of consumption in the two sectors are set at 0.61 with a standard error of 0.075, which is close to the estimates of Liang and Li (2011); the inverse elasticity of labor supply is common to both household types, following a Gamma distribution with mean 0.5 and standard deviation 0.1, and the elasticity of substitution across hours for both households has a Normal (1, 0.1) prior distribution. In production function, the Calvo parameters for price setting, following Liu (2008) is set to 0.85 as a Beta distribution, whereas the Calvo parameters for wage setting in both sectors are set around 0.6. The indexation parameters are distributed according to a Beta (0.5, 0.2) prior distribution. Finally, we set the parameters capturing the income share of patient household as a Beta distribution with parameters 0.65 and 0.05 suggested by Xiao and Peng (2011).

Table 3.2 Prior Distribution of the Parameters

Parameter	Distribution	Mean	Standard Deviation
ε	Beta	0.61	0.075
$\acute{\varepsilon}$	Beta	0.61	0.075
α	Beta	0.65	0.05
θ_{π}	Beta	0.85	0.05
θ_{wc}	Beta	0.6	0.05
θ_{wh}	Beta	0.6	0.05
ι_{π}	Beta	0.5	0.2
ι_{wc}	Beta	0.5	0.2
ι_{wh}	Beta	0.5	0.2
η	Gamma	0.5	0.1
$\acute{\eta}$	Gamma	0.5	0.1
ξ	Normal	1	0.1

Table 3.2 Prior Distribution of the Parameters (Continued)

Parameter	Distribution	Mean	Standard Deviation
$\acute{\xi}$	Normal	1	0.1
r_R	Beta	0.75	0.1
r_Y	Normal	0	0.1
r_π	Normal	1.63	0.05
ρ_g	Beta	0.8	0.1
ρ_{ah}	Beta	0.8	0.1
ρ_{ac}	Beta	0.8	0.1
ρ_{ak}	Beta	0.8	0.1
ρ_z	Beta	0.8	0.1
ρ_j	Beta	0.8	0.1
ρ_τ	Beta	0.8	0.1
ρ_e	Beta	0.8	0.1
ρ_h	Beta	0.8	0.1
σ_z	Uniform	3	1.7
σ_j	Uniform	3	1.7
σ_τ	Uniform	3	1.7
σ_{ac}	Uniform	3	1.7
σ_{ak}	Uniform	3	1.7

Table 3.2 Prior Distribution of the Parameters (Continued)

Parameter	Distribution	Mean	Standard Deviation
σ_{ah}	Uniform	3	1.7
σ_p	Uniform	3	1.7
σ_e	Uniform	3	1.7
σ_R	Uniform	3	1.7
σ_{pl}	Uniform	3	1.7
σ_g	Uniform	3	1.7
σ_h	Uniform	3	1.7

3.6 Posterior Distributions

After deriving the likelihood function and specifying the priors, we can then estimate the posterior distribution, $P(\theta|y^*)$, which denotes the probabilities for different values of the parameters after observing the data. It can be considered as an update of the probabilities given by the prior, based on the additional information provided by the variables in our data. According to the Bayes theorem, the posterior distribution is shown as :

$$p\left(\theta \middle| y^*\right) = \frac{p\left(\theta, y^*\right)}{p\left(y^*\right)} = \frac{p\left(y^* \middle| \theta\right) p\left(\theta\right)}{p\left(y^*\right)} \tag{3.51}$$

where $p(\theta, y^*)$ represents the joint density of the parameters and the data, $p(y^*|\theta)$ is the density of the data conditional on the parameters (the likelihood), $p(\theta)$ denotes the prior distribution of parameters and $p(y^*)$ is the marginal density of the data. Note that $p(y^*)$ is independent with θ and thus can be treated as a constant for the estimation, so we get:

$$p\left(\theta \middle| y^*\right) \propto p\left(y^* \middle| \theta\right) p(\theta) = K\left(\theta \middle| y^*\right) \tag{3.52}$$

where $K(\theta|y^*)$ is the posterior kernel, proportional to the posterior by $p(y^*)$. Next, we take logs for both side of the equation and get that:

$$\ln K\left(\theta \middle| y^*\right) = \ln p\left(y^* \middle| \theta\right) + \ln p(\theta) = \ln L\left(\theta \middle| y^*\right) + \sum_{x=1}^{n} \ln p\left(\theta_x\right) \tag{3.53}$$

where n is the number of parameters being estimated and priors are assumed to be independent.

The kernel equation is nonlinear and complicated function of the structural θ, and we cannot obtain an explicit form for it. So the sampling-like methods such as Monte Carlo Markov Chain can be used.

3.6.1 Monte Carlo Markov Chain (MCMC)

Monte Carlo Markov Chain (MCMC) methods are simulation techniques that generate a sample from some target distribution. The Markov Chain is a stochastic process in which future states are dependent of past states given the present state and Monte Carlo is a simulation process by computer. The idea of MCMC is to appoint a transition kernel for a Markov chain and then begin with some initial value and iterate a number of times, and produce a limiting distribution which is the objective distribution we need to sample from.

There are generally two algorithms of MCMC in Bayesian analysis: the Gibbs Sampler and the Metropolis-Hastings algorithm. In this study, we only focus on Metropolis-Hastings (MH) algorithm, because Gibbs Sampling has a lot of limit and assumption in comparison with MH algorithm

The MH algorithm procedure is followed (Lam 2010):

First step is to assume that the posterior is $P(\theta|y)$, then choose a starting value θ^0. This is equivalent to drawing from our initial stationary distribution and θ^0 must have positive probability.

Second, a candidate θ^* is drew from a jumping distribution $J_t(\theta^*|\theta^{t-1})$ at iteration t. The jumping distribution $J_t(\theta^*|\theta^{t-1})$ determines Markov chain in next iteration. If the jumping distribution is dependent on θ^{t-1}, then we have a so called random walk Metropolis sampling. If the jumping distribution does not depend on θ^{t-1},

$$J_t\left(\theta^*\middle|\theta^{t-1}\right) = J_t\left(\theta^*\right) \tag{3.54}$$

then we have what is known as independent Metropolis-Hastings sampling.

Third step is to compute an acceptance ratio (probability):

$$r = \frac{p\left(\theta^*|y\right)/J_t\left(\theta^*\middle|\theta^{t-1}\right)}{p\left(\theta^{t-1}|y\right)/J_t\left(\theta^{t-1}\middle|\theta^*\right)} \tag{3.55}$$

If the candidate draw has higher probability than the current draw, then the candidate is better so we definitely accept it. Otherwise, the candidate is accepted according to the ratio of the probabilities of the candidate and current draws.

The fourth step is that accepting θ^* as θ^t with probability min (r, 1). If θ^* is not accepted, then $\theta^t = \theta^{t-1}$. For each θ^*, draw a value u from the Uniform(0, 1) distribution. If $u \leq r$, then accept θ^* as θ^t. Otherwise, use θ^{t-1} as θ^t. Candidate draws with higher density than the current draw are always accepted.

Finally, we repeat steps 2-4 M time to get M draws from $P(\theta|y)$.